Date 27/05/2020

D1 — Mathematics

Topic - characteristic roots and ch. Vector

By Dr. Ram Anek Yadav, Deptt. of Maths, Marwari College, Darbhanga.

Question 1. Find the eigen values and corresponding eigen vectors for the matrix

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

<u>Solution</u>: characteristic equation is

$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 3-\lambda & 1 & 4 \\ 0 & 2-\lambda & 6 \\ 0 & 0 & 5-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (\lambda-3)(\lambda-2)(\lambda-5) = 0$$

$$\therefore \lambda = 2, 3, 5$$

characteristic vector is given by

$$(A - \lambda I)x = 0$$

$$\Rightarrow \begin{bmatrix} 3-\lambda & 1 & 4 \\ 0 & 2-\lambda & 6 \\ 0 & 0 & 5-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (1)$$

ch. vector corresponding to ch. value $\lambda = 5$ is given by

$$\begin{bmatrix} -2 & 1 & 4 \\ 0 & -3 & 6 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Date 27/05/2020

D1 – Mathematics

By Dr. R.A. Yadav

Rank $A = 2$ = the number of non zero rows $r < n$

$\therefore n - r = 1$

Variable will be assigned arbitrary value and the remaining two will be found in terms of this arbitrary value. The equivalent system of equations are

$-2x_1 + x_2 + 4x_3 = 0$,

$-3x_2 + 6x_3 = 0$

& choose $x_3 = k$

$\Rightarrow -3x_2 + 6k = 0$

$\Rightarrow -3x_2 = -6k$

$\Rightarrow x_2 = 2k$

Now putting the value of x_2 & x_3, we get

$-2x_1 + 2k + 4k = 0$

$\Rightarrow -2x_1 + 6k = 0$

$\Rightarrow -2x_1 = -6k$

$\Rightarrow x_1 = 3k$.

$\therefore$ characteristic vectors are $3k$, $2k$ and k

i.e. $X = \{3k, 2k, k\}$

For $\lambda = 2, 3$ there will be only trivial solution as rank $A = 3$ and hence there will be non zero characteristic vector corresponding to these values of λ.