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B - Mathematics

Topic - Linear Equation

Paper - I

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(1) Find the matrix X from the equation $AX = B$, where

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\& \; B = \begin{bmatrix} 2 \\ 1 \\ 7 \end{bmatrix}$$

<u>Solution</u>: Let the matrix X be $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

$$\therefore C = [A, B] = \begin{bmatrix} 1 & -1 & 0 & 2 \\ 0 & 1 & -1 & 1 \\ 1 & 1 & 1 & 7 \end{bmatrix}$$

$$C \sim \begin{bmatrix} 1 & -1 & 0 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 2 & 1 & 5 \end{bmatrix} \text{ by } R_3 - R_1$$

$$\text{or } C \sim \begin{bmatrix} 1 & -1 & 0 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 3 & 3 \end{bmatrix} \text{ by } R_3 - 2R_1$$

The equivalent system of equations are

$$x - y = 2, \quad y - z = 1, \quad 3z = 3$$

$$\therefore z = 1, \; y = 2 \; \& \text{ hence } x = 4.$$

$$\therefore X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix}$$

(2) Solve by matrix method
$x + 2y + 3z = 14$, $3x + y + 2z = 11$ & $2x + 3y + z = 11$.

<u>Solution</u>: the given set of equation can be written as $AX = B$.

i.e. $$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 14 \\ 11 \\ 11 \end{bmatrix}$$

$$C = [A, A] = \begin{bmatrix} 1 & 2 & 3 & 14 \\ 3 & 1 & 2 & 11 \\ 2 & 3 & 1 & 11 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 & 14 \\ 0 & -5 & -7 & -31 \\ 0 & -1 & -5 & -17 \end{bmatrix}$$ by $R_2 - 3R_1$ and $R_3 - 2R_1$.

$$\sim \begin{bmatrix} 1 & 2 & 3 & 14 \\ 0 & 1 & 5 & 17 \\ 0 & 5 & 7 & 31 \end{bmatrix}$$ interchanging R_2 & R_3 and then $R_2(-1)$ & $R_3(-1)$

$$\sim \begin{bmatrix} 1 & 2 & 3 & 14 \\ 0 & 1 & 5 & 17 \\ 0 & 0 & -18 & -54 \end{bmatrix}$$ by $R_3 - 5R_1$

Clearly rank $\{A, A\} = 3$, the number of non zero rows. Also $\rho(A) = 3$. $\because \rho(A, A) = \rho(A) = 3$ the number of variables. therefore the eqn of system are consistent and the solution is unique. The equivalent system of eqn are
$x + 2y + 3z = 14$, $y + 5z = 17$ & $-18z = -54$
$\Rightarrow z = 3$, $y = 2$ and hence $x = 1$.