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D_1 – Mathematics

Topic – Linear Equation

By Dr. Ram Anek Yadav, Dept. of Maths, Marwari College, Darbhanga.

problem 1. Discuss for all values of k the system of equations

$$2x + 3ky + (3k+4)z = 0$$

$$x + (k+4)y + (4k+2)z = 0 \quad \text{or } AX = 0$$

$$x + 2(k+1)y + (3k+4)z = 0$$

<u>Solution</u>: Equivalent system of equations can be put in the form of $AX = 0$

i.e. $$\begin{bmatrix} 2 & 3k & 3k+4 \\ 1 & k+4 & 4k+2 \\ 1 & 2k+2 & 3k+4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

Here $A \sim \begin{bmatrix} 1 & k+4 & 4k+2 \\ 2 & 3k & 3k+4 \\ 1 & 2k+2 & 3k+4 \end{bmatrix}$ by $R_2 - 2R_1$, $R_3 - R_1$

$$\sim \begin{bmatrix} 1 & k+4 & 4k+2 \\ 0 & k-8 & -3k \\ 0 & k-2 & -k+2 \end{bmatrix}$$

Let us evaluate $|A|$.

$$|A| = 1\begin{vmatrix} k-8 & -5k \\ k-2 & -k+2 \end{vmatrix}$$

$$= (k-8)(-k+2) + (k-2)\cdot 5k$$

$$= -k^2+8k+2k-16+5k^2-10k$$

$$= 4k^2-16$$

Now $|A| = 0$

$\Rightarrow 4k^2-16=0 \Rightarrow 4k^2=16 \Rightarrow k^2=4 \Rightarrow k=\pm 2$

For $k=2$ the equivalent system of eq^{n}s be

$$\begin{bmatrix} 1 & 6 & 10 \\ 0 & -6 & -10 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$\Rightarrow x+6y+10z=0, \quad -6y-10z=0, \quad 0=0$

choose $z=c$, we get

$y = -\frac{5c}{3}$ & $x=0$

For $k=-2$ equivalent system of eq^{n}s be

$$\begin{bmatrix} 1 & 2 & -6 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

In this case by proceeding similarly, we get
$x+2y-6z=0$ & $y-z=0, \quad 0=0$
For $z=k, \; y=k \quad \therefore x=4k.$