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Dr- Mathematics

Topic - Laplace Transform

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Ex. 1. Find $L\{e^{-t}(3\sinh 2t - 5\cosh 2t)\}$.

Solution: We have $L\{3\sinh 2t - 5\cosh 2t\}$

$$= 3L\{\sinh 2t\} - 5L\{\cosh 2t\}$$

$$= 3 \cdot \frac{2}{p^2-2^2} - 5 \cdot \frac{p}{p^2-2^2}$$

$$= \frac{6}{p^2-4} - \frac{5p}{p^2-4}$$

$$= \frac{6-5p}{p^2-4} = F(p)$$

Hence from the first shifting theorem,

$$L\{e^{-t}(3\sinh 2t - 5\cosh 2t)\} = F(p+1)$$

$$= \frac{6-5(p+1)}{(p+1)^2-4} = \frac{1-5p}{p^2+2p+1-4}$$

$$= \frac{1-5p}{p^2+2p-3}.$$

Ex. 2:- Find $L\{f(t)\}$ where

$$f(t) = \begin{cases} \cos\left(t - \frac{2}{3}\pi\right), & t > \frac{2}{3}\pi \\ 0, & t < \frac{2}{3}\pi \end{cases}$$

<u>Solution</u>: Let $\varphi(t) = \cos t$

$$\therefore f(t) = \begin{cases} \cos\left(t - \frac{2}{3}\pi\right) = g\left(t - \frac{2}{3}\pi\right), & t > \frac{2}{3}\pi \\ 0, & t < \frac{2}{3}\pi \end{cases}$$

We have $L\{\varphi(t)\} = L\{\cos t\}$.

$$= \frac{p}{p^2+1} = F(p)$$

Hence from the second shifting theorem, we have

$$L\{f(t)\} = e^{\left(-\frac{2\pi}{3}\right)p} \cdot F(p)$$

$$= e^{-\frac{2\pi p}{3}} \cdot \frac{p}{p^2+1}.$$