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For B.Sc. Part-I

Book: Theory of Equation

Subject Mathematics

Lecture on Newton - Vieta method by

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Newton - Vieta method :- We want to determine an approximate real number p of the polynomial equation

$$f(x) = x^n + a_1 x^{n-1} + a_2 x^{n-2} + \cdots + a_{n-1}x + a_n = 0 \quad (1)$$

where $a_1, a_2, a_3, \ldots a_n$ are real numbers.

Let $x^n + a_1 x^{n-1} + a_2 x^{n-2} + \cdots + a_{n-1}x + a_n$

$$= (x-p)(x^{n-1} + b_1 x^{n-2} + \cdots + b_{n-2}x + b_{n-1}) R$$

where R is the remainder.

Comparing the like powers of x, we get

$a_1 = b_1 - p \Rightarrow b_1 = a_1 + p$

$a_2 = b_2 - pb_1 \Rightarrow b_2 = a_2 + pb_1$

$a_3 = b_3 - pb_2 \Rightarrow b_3 = a_3 + pb_2$

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$a_k = b_k - pb_{k-1} \Rightarrow b_k = a_k + pb_{k-1}$

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$a_{n-1} = b_{n-1} - pb_{n-2} \Rightarrow b_{n-1} = a_{n-1} + pb_{n-2}$

$a_n = R - pb_{n-1} \Rightarrow R = a_n + pb_{n-1} \quad (2)$

We introduce a quantity b_n and define the recurrence relations $b_k = a_k + pb_{k-1}$; $k = 1, 2, 3, \ldots, n$, $b_0 = 1 \quad (3)$

Then $b_n = a_n + p b_{n-1} = R$

Therefore from $f(p) = R$

Therefore $f(p) = R = b_n$ ——— ④

Now from Newton – Raphson formulae, we have for the first approximation

$$p_1 = p_0 - \frac{f(p_0)}{f'(p_0)} = p_0 - \frac{f(p_0)}{(db_n/dp)_{p=p_0}} \quad \text{from ④} \quad —⑤$$

Now, we have to simplify eqn ⑤.

Differentiating ③ w.r.t. p, we get

$$\frac{db_k}{dp} = 0 + \left(b_{k-1} \cdot 1 + p\frac{db_{k-1}}{dp}\right) = b_{k-1} + p\frac{db_{k-1}}{dp} \quad —⑥$$

Let us write $\frac{db_k}{dp} = c_{k-1}$

Then from ⑥ We have $c_{k-1} = b_{k-1} + p c_{k-2}$, $k = 1, 2, -(n-1)$

Thus c_k is determined from b_k in the same way as b_k is determined from a_k.

Now $f(p) = b_n$ and $f'(p) = \frac{db_n}{dp} = c_{n-1}$

Hence from ⑤ Can be written as

$$p_1 = p_0 - \frac{b_n}{c_{n-1}}$$

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Lecture on Solution of numerical equation by Newton's Raphson Method. by Dr. Ram Anek Yadav, Assistant Guest Professor, Deptt of Maths, MarWari College, Darbhanga.

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Problem (1): Solve the equation $x^3 - x - 1 = 0$ correct to three decimal places.

Solution: Given numerical eqn is $x^3 - x - 1 = 0$.

Complete equation is

$$x^3 + 0x^2 - 1.x - 1 = 0 \quad (1)$$

1	1	0	-1	-1
		1	1	0
	1	1	0	$-1 = b_n$
1		1	2	
	1	2	$2 = c_{n-1}$	

As a first approximation to the root near $x = 1$, We have

$$P_1 = P_0 - \frac{b_n}{c_{n-1}} = 1 - \frac{(-1)}{2} = 1 + \frac{1}{2} = \frac{3}{2} = 1.5$$

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For the Second approximation:

1.5	1	0	-1	-1
		1.5	2.25	1.875
	1	1.5	1.25	$0.875 = b_n$
1.5		1.5	45	
	1	·3	$5.75 = c_{n-1}$	

Therefore as a second approximation, we have

$$P_2 = 1.5 - \frac{0.875}{5.75} = 1.5 - 0.152 = 1.348$$
$$= 1.35 \text{ (approximately)}$$

For the third approximation:-

1.35	1	0	-1	-1
		1.35	1.822	1.110
	1	1.35	0.822	0.11
1.35		1.35	3.645	
	1	2.70	4.467	

Therefore as a third approximation, we have

$$P_3 = 1.35 - \frac{.11}{4.467} = 1.35 - 0.024 = 1.326 \text{ (nearly)}$$