Date 08/05/2020

D1 – Mathematics

Topic – Determinant

Paper – I.

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Q Prove that

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ a & b & c & d \\ a^2 & b^2 & c^2 & d^2 \\ a^3 & b^3 & c^3 & d^3 \end{vmatrix} = (a-b)(a-c)(a-d)(b-c)(b-d)(c-d)$$

<u>Solution</u>: putting $a=b$ in given determinant, C_1 and C_2 become identical. Then $\Delta=0$. Hence $a-b$ is a factor. Again putting $a=c$ in given determinant, C_1 and C_3 are identical. Hence $\Delta=0 \Rightarrow (a-c)$ is also a factor of Δ. Also putting $a=d$, we get C_1 & C_4 become identical. Hence $\Delta=0$. $\therefore (a-d)$ is also a factor of Δ. putting $b=c$ in given determinant we get C_2 & C_3 are identical. Hence $\Delta=0$. Therefore $(b-c)$ is a factor of Δ. putting $b=d$, C_2 & C_4 are identical. Hence $\Delta=0$. $\therefore (b-d)$ is a factor of Δ. putting $c=d$ we have C_2 & C_4 are identical. Hence $\Delta=0$. Hence $(c-d)$ is a factor.

$\therefore \Delta = (a-b)(a-c)(a-d)(b-d)(b-c)(c-d)$.

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Problem 2: prove that

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & zx & xy \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$$

Solution: Multiplying C_1, C_2 & C_3 by x, y, z respectively we get

$$\Delta = \frac{1}{xyz}\begin{vmatrix} x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \\ xyz & xyz & xyz \end{vmatrix} = \frac{\cancel{xyz}}{\cancel{xyz}}\begin{vmatrix} x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \\ 1 & 1 & 1 \end{vmatrix}$$ by taking xyz as common from R_3

$$= (-1)^2\begin{vmatrix} 1 & 1 & 1 \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 1 \\ x^2-y^2 & y^2-z^2 & z^2 \\ x^3-y^3 & y^3-z^3 & z^3 \end{vmatrix}$$ By $C_1 - C_2$ & $C_2 - C_3$

$$= 1\begin{vmatrix} x^2-y^2 & y^2-z^2 \\ x^3-y^3 & y^3-z^3 \end{vmatrix} = (x^2-y^2)(y^3-z^3) - (y^2-z^2)(x^3-y^3)$$

$$= (x+y)(x-y)\{(y-z)(y^2+yz+z^2) - (y+z)(y-z)\{(x-y)(x^2+xy+y^2)\}$$

$$= (x-y)(y-z)\left[(x+y)(y^2+yz+z^2) - (y+z)(x^2+xy+y^2)\right]$$

$$= (x-y)(y-z)\left[\cancel{xy^2} + \cancel{xyz} + xz^2 + \cancel{y^3} + \cancel{y^2z} + yz^2 - yx^2 - \cancel{xy^2} - \cancel{y^3} - zx^2 - \cancel{xyz} - \cancel{zy^2}\right]$$

$$= (x-y)(y-z)\left[(z-x)(xy+yz+zx)\right]$$

$$= (x-y)(y-z)(z-x)(xy+yz+zx)$$ proved